ponents. Their amplitude is small and the period is larger than the entry length. These disturbances are not found in the absence of the Hall and ion slip currents. This increases the entry length. This conclusion agrees with the results of Saric and Touryan.[6]

Numerically, the entry length has been calculated on the basis of 5% deviations in the fully developed centerline velocity. The entry lengths and K_{fd}, the fully developed values of the correction term K due to entrance effects in the pressure distribution are shown in Table 1.

The entry length for the case of no Hall and ion slip currents with Hartmann number $H_a = 10$ was found to be 0.0143 $R_e h$ by Chen and Chen.[4] With the Hall parameter $\beta_e = 2.0$ it is 0.0207 $R_e h$ and with $\beta_e = 2.0$ and ionslip parameter $\beta_i = 0.5$, it is found to be 0.0216 $R_e h$. As Hartmann number increases the entrace length decreases sharply.

The pressure distribution in the entrance region is shown in Fig. 1 for different values of H_a, β_e, and β_i. It shows small curvature in the inlet region. For the downstream region it becomes constant. The solutions presented herein may be useful for the investigation of the stability of a developing MHD flow with Hall and ion slip currents.

Acknowledgements

The authors thank referees for bringing to their attention Ref. 6. This work was carried out for a research project sponsored by Council of Scientific and Industrial Research, India.

References

[1]Slezkin, N. A., *Dynamics of Viscous Incompressible Fluids*, Gostekhizdat, Moscow, 1965, (in Russian).

[2]Sparrow, E. M., Lin, S. H., and Lundgren, T. S., "Flow Development in the Hydrodynamic Entrance Region of Tubes and Ducts," *The Physics of Fluids*, Vol. 7, 1964, p. 338.

[3]Snyder, W. T., "Magnetohydrodynamic Flow in the Entrance Region of a Parallel Plate Channel," *AIAA Journal*, Vol. 3, Oct. 1965, p. 1833.

[4]Chen, T. S. and Chen, G. L., "MHD Channel Flow with an Arbitrary Inlet Velocity Profiles," *The Physics of Fluids*, Vol. 15, 1972, p. 1531.

[5]Hwang, C. S., "Linearized Analysis of MHD Channel Entrance Flow," *The Physics of Fluids*, Vol. 15, 1972, p. 1852.

[6]Saric, W. S. and Touryan, K. J., "Incompressible Magnetohydrodynamic Entrance Flow in Plane Channel," *The Physics of Fluids*, Vol. 12, 1969, p. 1412.

[7]Sherman, A. and Sutton, G.W., *Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965.

[8]Zhaveri, V., "Influence of Hall Effect and Ionslip Effect on Velocity and Temperature Fields in a MHD Channel," *Wärme-und Stoffübertragung*, Vol. 7, 1974, p. 226.

[9]Mittal, M. L. and Bhat, A. N., *Proceedings of Symposium in Plasma Physics and Magnetohydrodynamics*, 1975, Bhabha Atomic Research Center, India, p. 1.

Correlation for Gasdynamic Laser Gain

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Introduction

GASDYNAMIC lasers (GDL's) have spearheaded a breakthrough in high energy laser technology. These devices are essentially supersonic wind tunnels which create a lasing medium by rapid expansion of a vibrationally excited molecular gas. Under suitable conditions, laser power can be extracted from the supersonic stream. The physical processes, practical importance, experimental data, and theoretical analyses associated with GDL's are described in a recent book.[1]

Theoretical prediction of gasdynamic laser performance generally requires a sophisticated nonequilibrium nozzle flow computer program, such as Ref. 2. Moreover, GDL performance is a function of numerous variables, such as nozzle shape and size, reservoir gas temperature and pressure, mixture ratio, etc. For these reasons, an analysis of gasdynamic laser characteristics is usually restricted to those individuals and organizations which have considerable computational capability. Therefore, there is a need for engineering correlations which allow quick and easy hand calculations of GDL performance without a gross compromise in accuracy. The purpose of the present Note is to provide such an engineering correlation. In particular, a formula is provided for the calculation of peak small-signal gain for CO_2-N_2-H_2O gasdynamic lasers. Such a mixture is common in practical GDL field and laboratory devices.

The Correlation

Small-signal gain, G_o, is a measure of the amplifying property of the laser medium such that $dI_\nu = G_o I_\nu dx$, where I_ν is the laser beam intensity, and dI_ν is the increase in intensity over a distance dx. Essentially, G_o is a negative absorption coefficient. In gasdynamic lasers, G_o varies with distance in the flow direction, first increasing, reaching a peak, then decreasing downstream.[1] In the present Note, the peak value of G_o is correlated as the following functional variation,

$$G_o = f(P_o, T_o, A_e/A^*, h^*, X_{N_2}, X_{CO_2}, X_{H_2O}) \tag{1}$$

where P_o and T_o are the reservior gas pressure and temperature, respectively, A_e/A^* is the nozzle exit-to-throat area ratio, h^* is the throat height, and X_i is the mole fraction of species i in the mixture.

Using the computer program described in Ref. 2, with updated kinetic rates as given in Ref. 3, a large number of parametric variations were obtained. The parameters were those of Eq. (1), with the exception that P_o and h^* were grouped as the product $P_o h^*$, based on binary scaling, and $X_{CO_2} + X_{N_2} + X_{H_2O} = 1$. This large bulk of numerical data was then systematically correlated, using a combination of chi-square and Gaussian functions, aided by least-squares fits. The details are given in Ref. 4. The resulting engineering formula, although rather long, can easily be evaluated on a slide rule or pocket calculator. The formula is

$$G_o = K_O - G[(1-Q1) + (1-Q2) + (1-Q3)] - Q4 \tag{2}$$

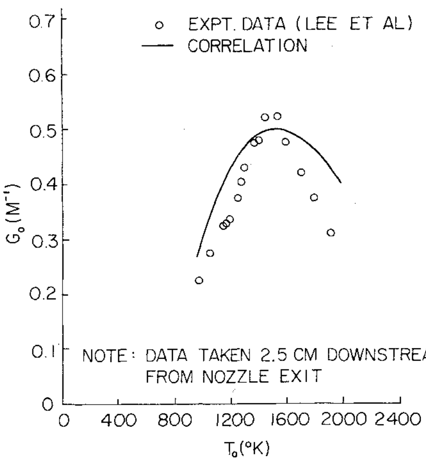

Fig. 1 Comparison of Eq. (2) with data of Ref. 5. $h^* = 0.1$ cm, $A_e/A^* = 15$ ($M=4$), $P_o = 9$ atm, $\%H_2O = 2.4$, $\%CO_2 = 6.3$.

Received May 20, 1976.
Index categories: Reactive Flows; Lasers.

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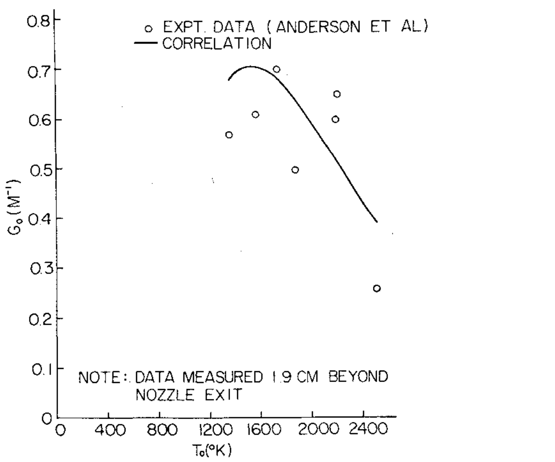

Fig. 2 Comparison of Eq. (2) with data of Ref. 6. $h^* = 0.1$ cm, $A_e/A^* = 20$, $P_o = 10.23$ atm, $\%H_2O = 1.2$, $\%CO_2 = 10.3$.

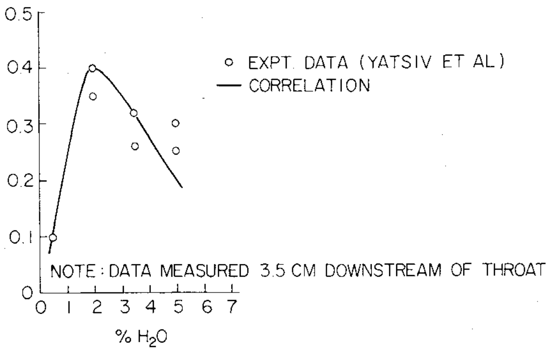

Fig. 3 Comparison of Eq. (2) with data of Ref. 7. $h^* = 0.13$ cm, $A_e/A^* = 15$ (wedge nozzle with 15° half-angle), $P_o = 7.5$ atm, $T_o = 1650$K, $\%CO_2 = 20$.

where

$$K_O = 1.484\, e^{-(A_e/A^* - 53.037)^2/1532.3}$$
$$G = 0.875\, K_O$$
$$Q1 = 0.025\, T_X^{2.673}\, e^{-0.2475 T_X}$$
$$Q2 = 2.75 \sin(C_X)\, e^{-C_X/1.171}$$
$$Q3 = 0.8\,(H_2O^1 + 1.0)^{0.65}\, e^{-0.245 H_2O^1}$$

$$Q4 = 0.1655 \exp\left[\frac{-(A_e/A^* - 39.15)^2}{176.72}\right](P_0 h^*)^{3/2}$$

$$T_X = [T_O/(1.0 - .105AF) - 470.]/100.0$$
$$C_X = (\%CO_2 \pi)/(40. + 11.AF)$$
$$H_2O^1 = (\%H_2O - CA)(\%CO_2/24 - CA/3.3)(1.05 - 0.35AF) + 1.65$$
$$CA = 2.25 - 2.0\, e^{-\%CO_2/10.0}$$
$$AF = (A_e/A^* - 19.75)/30.25$$

T_O = stagnation temperature (K)
$\%CO_2$ = mole fraction of CO_2
$\%H_2O$ = mole fraction of H_2O
P_O = stagnation pressure (atm)
h^* = throat height (cm)
A_e/A^* = nozzle exit area to throat area ratio

Equation (2) is valid only for minimum-length nozzles, $A_e/A^* < 53$, $0.5 < \%H_2O < 30$, $\%CO_2 < 30$ and $P_o h^* < 3.0$. However, these ranges include almost all cases of practical interest.

Results

As shown in Ref. 4, Eq. (2) correlates the numerical data of Refs. 2 and 3 within 1 or 2% for most of the runs made.

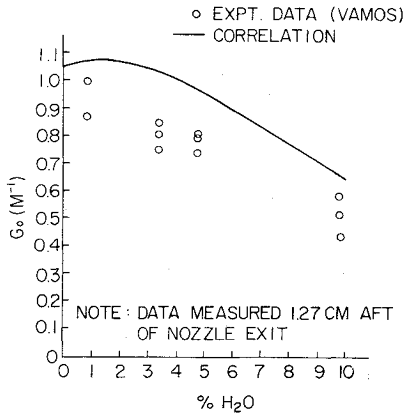

Fig. 4 Comparsion of Eq. (2) with data of Ref. 8. $h^* = 0.0356$ cm, $A_e/A^* = 50$, $P_o = 37.5$ atm, $T_o = 1800$K, $\%CO_2 = 7$.

However, the real test of the formula is its application to experimental conditions. Figures 1-4 show such a comparison, where Eq. (2) is matched with the experimental data of Lee et al.,[5] Anderson et al.,[6], Yatsiv et al.,[7] and Vamos,[8] respectively. The data are chosen because they encompass a wide range of area ratio, temperature, and mixture ratio. In all cases, the comparison is reasonable. However, emphasis is made that the correlation is for peak G_o, whereas the experimental data are obtained at fixed locations in the laser cavity which do not necessarily correspond to the location of peak gain. Also, Eq. (2), being based on the numerical data of Refs. 2 and 3, is therefore subject to the same shortcomings of the theoretical analysis and kinetic rates. Nevertheless, the correlation appears to be reasonalby accurate. This is particularly remarkable considering the complicated dependency of G_o on the parameters given in Eq. (1).

Conclusions

A closed-form correlation is given for the peak small-signal gain in CO_2-N_2-H_2O gasdynamic lasers. This is the first step towards quick and easy "back-of-the-envelope" estimates of GDL performance. The correlation agrees reasonalby well with experimental data. In view of the fact that the correlations encompasses a large number of GDL parameters over wide ranges, such agreement is rather remarkable.

References

[1] Anderson, J. D., Jr., *Gasdynamic Lasers: An Introduction*, Academic Press, New York, 1976.

[2] Glowacki, W. J. and Anderson, J. D., Jr., "A Computer Program for CO_2-N_2-H_2O Gasdynamic Laser Gain and Maximum Available Power," NOLTR 71-210, Naval Ordnance Lab., White Oak, Md., Oct., 1971.

[3] Anderson, J. D., Jr., "Effect of Kinetic Rate Uncertainties on Gasdynamic Laser Gain and Energy Predictions," *AIAA Journal*, Vol. 13, Dec., 1974, pp. 1699-1703.

[4] McManus, J. I., "Correlation Equation for Peak Small Signal Gain of Continuous Wave CO_2-N_2-H_2O Gasdynamic Lasers," Master's Thesis, Dept. of Aerospace Engineering, Univ. of Maryland, College Park, Md., 1976.

[5] Lee, G., Gowen, F. E. and Hagen, J. R., "Gain and Power of CO_2 Gasdynamic Laser," *AIAA Journal*, Vol. 10, Jan., 1972, pp. 65-71.

[6] Anderson, J. D., Jr., Humphrey, R. L., Vamos, J. S., Plummer, M. J., and Jensen, R. E., "Population Inversions in an Expanding Gas: Theory and Experiment," *Physics of Fluids*, Vol. 14, Dec., 1971, pp. 2620-2624.

[7] Yatsiv, S., Greenfield, E., Dothan-Deutsch, F., Chuchem, D. and Bin-Nun, E., "Pulsed CO_2 Gas-Dynamic Laser," *Applied Physics Letters*, Vol. 19, Aug., 1971, pp. 65-68.

[8] Vamos, J. S., "Small-Signal Gain Measurements in a High-Ratio Nozzle Shock Tunnel GDL," AIAA Paper, 74-177.